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Introducing a New High-Order Chaotic System with an Equilibrium Point and Stabilizing It Using LQR Controller

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Abstract *–In this paper, a new high-order chaotic system is proposed. This system has an equilibrium point on center and its specific feature is existence of two large Lyapunov exponents compared to other high-order chaotic systems. In order to prove the existence of high-order chaos in this system, criteria such as energy dissipation of the system, instability of equilibrium point, system absorption and Lyapunov exponents of system are used. Investigating the mentioned criteria confirms the existence of chaos in the system under study. Then, by changing system parameters, different dynamic behaviors such as the limit cycle and high-order chaos can be observed in the system. Finally, using a Linear Quadratic Regulator (LQR) controller, chaotic system's stability around equilibrium point is guaranteed.*

Keywords*: Chaotic System, High-Order Chaos, Lyapunov Exponent, Equilibrium Point, LQR Controller*

INTRODUCTION

Chaotic systems have attracted the attention of scientists in various fields in recent years [1, 2]. Chaos theory explores chaotic dynamical systems. These systems are dynamical and nonlinear in nature that are very susceptible to initial conditions; slight changes in the initial conditions of the system cause many changes in the future. Behavior of chaotic systems appears to be random, but there is no need for randomness in creating chaotic behavior and definite dynamical systems can show chaotic behavior too. Various methods such as linear and nonlinear feedback control and adaptive control are used to analyze the chaotic behavior of these systems.

The emergence of new mathematical and numerical tools has played an important role in understanding and describing the concept of chaos. These tools have been helpful to detect chaos in many scientific fields such as biology [3], chemistry [4] and engineering applications [5, 6]. Research on Chaotic systems can be divided into three general categories: review of new chaotic systems [7, 8], harmony in chaotic systems [9, 10] and the control of chaos [11]. Research done in this paper is placed in the third category. After the first research on the area of chaos control [12], efforts to control the chaotic systems are done with three main objectives. The first objective which is purely classical consists of stabilization of one of the unstable equilibrium points [13-16].

The second objective is using control strategy to achieve harmony in the system [17-19]. The third is to control the chaotic systems to stabilize unstable periodic paths in chaotic absorbents [20-23]. High-order chaos was first presented in [24]. High-order continuous chaotic systems have at least four state variables and the special characteristic of these systems is the existence of two positive Lyapunov exponents. This feature makes the dynamic of the system to extend in more than one direction simultaneously. One of the common methods to design a high-order chaotic system is to consider a low-order chaotic system with three state variables and adding a state feedback controller to it and retuning the system coefficients [25, 26]. In this case, the high-order chaos is created in the system.

In this paper, an LQR controller is applied to a nonlinear system that ensures system stability. The overall structure of this paper is as follows: In Section 2, after introducing the dynamics of high-order chaotic system, system energy dissipation and the instability of the equilibrium point are shown to prove chaos in the system. Then, system absorbent, time response and Lyapunov exponents of the system are investigated. In Section 3, the different dynamical behavior of the system can be observed by changing one of the system parameters. In Section 4, system is stabilized around its equilibrium point using an LQR controller design and Section 5 concludes the paper.

MATERIAL AND METHODS

2- Dynamical equations of the new high-order chaotic system

Dynamics equations of the high-order chaotic system studied above are derived by adding a fourth state variable and some nonlinear terms to low-order chaotic system is presented in [27]. System equations are given in formula 1.

$$\begin{cases} \dot{x} = a(y-x) + byz^2 = f_1(x,y,z,w) \\ \dot{y} = cx + dxz^2 + ew = f_2(x,y,z,w) \\ \dot{z} = fz + gy^2 + hxw = f_3(x,y,z,w) \\ \dot{w} = ky = f_4(x,y,z,w) \end{cases} \quad (1)$$

Where x, y, z and w represent the state variables of the chaotic system. By choosing the parameters as in (2), the system shows the behavior of a high-order chaotic system.

$$\begin{matrix} a = 7.7, & b = -1, & c = 8 \\ d = 4, & e = 8, & f = -4 \\ g = 1, & h = 1, & k = -2 \end{matrix} \quad (2)$$

Following conditions are necessary for the existence of chaos in a system.

• The system must be dissipative. Being dissipative means that the energy of the system should be reducing and the system must be inclusively stable.

• The system must have unstable equilibrium points. Jacobian matrix calculated in the equilibrium points must have unstable eigenvalues.

• System paths should be limited and bounded.

Next, these conditions will be reviewed.

2.1. Checking that if system is dissipative or not: Dynamical systems can be divided into two groups, conservative and dissipative. One of the necessary conditions for the existence of chaos in a system is that the system should be dissipative. To check that if system is dissipative or not, suppose that dynamic equations of the system are as follows,

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, \cdots, x_n) \\ \dot{x}_2 = f_2(x_1, x_2, \cdots, x_n) \\ \quad \vdots \\ \dot{x}_n = f_n(x_1, x_2, \cdots, x_n) \end{cases} \quad (3)$$

First   is calculated. If this value is equal to zero then the system is conservative and if this value is negative then the system is dissipative. Equation (4) provides the condition for system (1) to be dissipative considering the parameters in formula 2.

$$\nabla F = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial} + \frac{\partial f_3}{\partial z} + \frac{\partial f_4}{\partial 2} = -7.7 - 4 = -11.4 < 0 \quad (4)$$

Considering that the above expression is negative, system (1) is dissipative, though globally stable.

2.2. Evaluation of instability of system equilibrium point: To calculate the equilibrium point of the system is used which in this case only the equilibrium point of the system will be at. Jacobian matrix of the system at equilibrium point is obtained as follows.

$$J_{(0,0,0,0)} = \begin{bmatrix} -7.7 & 7.7 & 0 & 0 \\ 8 & 0 & 0 & 8 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \quad (5)$$

Eigenvalues of the Jacobian matrix is as follows.

$$\begin{cases} \lambda_1 = -12.2454 \\ \lambda_2 = 2.2727 + 2.2126j \\ \lambda_3 = 2.2727 + 2.2126j \\ \lambda_4 = -4 \end{cases} \quad (6)$$

According to the sign of the eigenvalues of Jacobian matrix of the system, it becomes clear that equilibrium point of the system is saddle point and thus unstable. In conclusion one can say that the system is locally unstable.

2.3. Absorbent of the high-order chaotic system: Simulating this high-order chaotic system in MATLAB, a number of absorbents of this system in two-dimensional and three-dimensional space are given in the form of Figures 1and 6. Initial conditions for simulation of chaotic system has been considered $(x_0, y_0, z_0, w_0) = (-4, -3, 1, 3)$

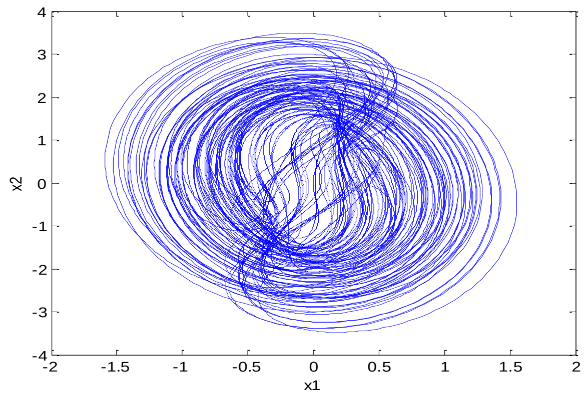

Figure 1. Absorbent of system in (x-y) space

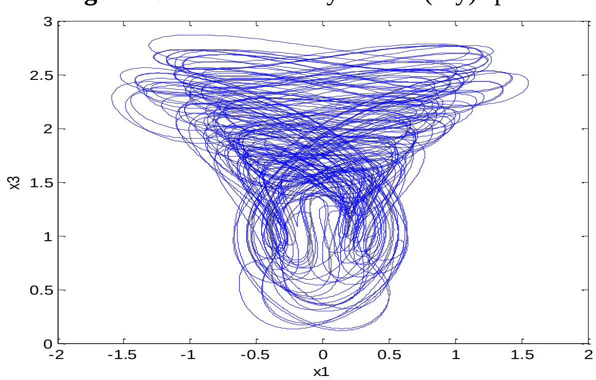

Figure 2. Absorbent of system in (x-z) space

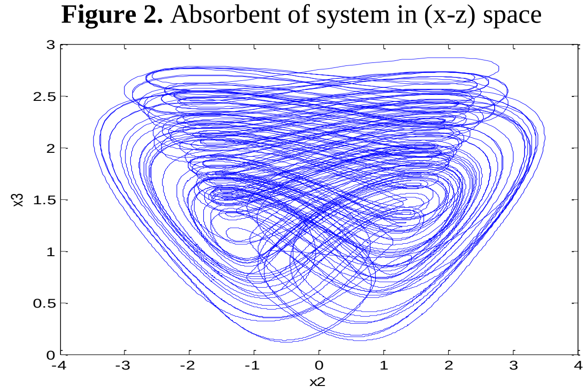

Figure 3. Absorbent of system in (y-z) space

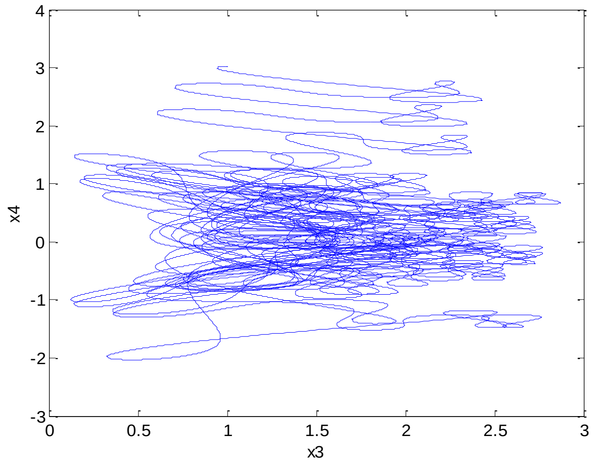

Figure 4. Absorbent of system in (z-w) space

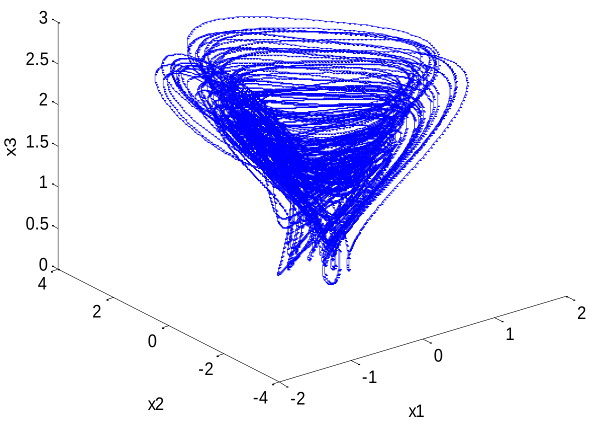

Figure 5. Absorbent of system in (x-y-z) space

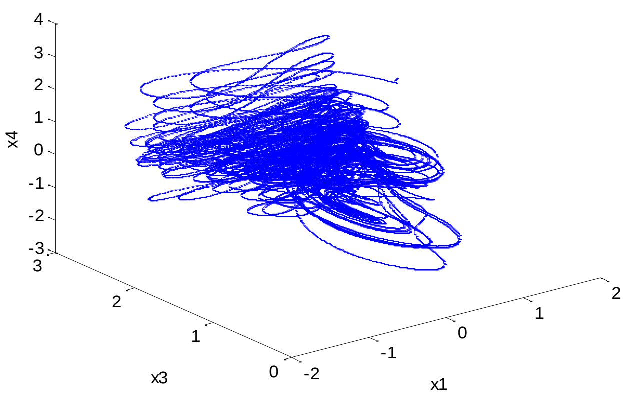

Figure 6. Absorbent of system in (x-z-w) space

2.4. Response time of state variables of high-order chaotic system: Figure 7 shows the response time of state variables of the high-order chaotic system. Initial conditions for simulation are considered as $(x_0, y_0, z_0, w_0) = (-4, -3, 1, 3)$.

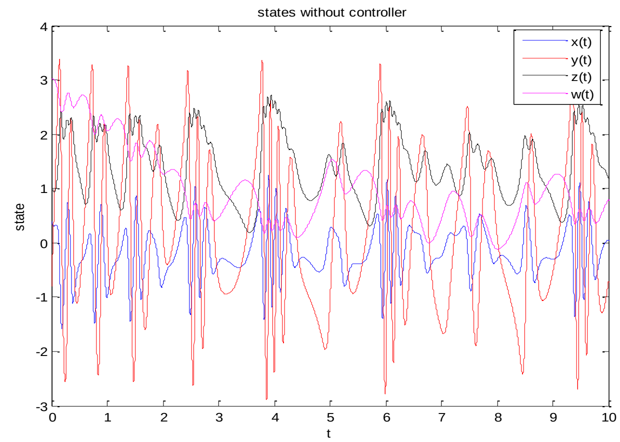

Figure 7. Response time of state variables of the high-order chaotic system

2.5. Evaluation of the Lyapunov exponent of high-order chaotic system: Lyapunov exponent was used in the year 1892 to control the stability of nonlinear differential equations. This method allows the study of the stability of differential equations without actually solving them. In order to call a system chaotic one must demonstrate that the system is highly dependent on the initial conditions. In other words, if two paths start at the initial conditions very close to each other, after a short period of time they diverged exponentially and take a completely different future. Lyapunov exponent specifies the dynamic sensitivity of the system to initial conditions. This quantity specifies the rate of convergence or divergence of two close paths in phase space. It is a standard quantity to determine whether a system is chaotic or not. For example, if the Lyapunov exponent is shown by λ, then

- If λ becomes positive then the distance between two points in the phase space increases exponentially, i.e. the system moves toward becoming chaotic.
- If λ becomes negative, one can conclude that the system shows stable behavior, in other words, the system reaches toward steady state.
- $\lambda = 0$ represents the boundary case.

Table 1 shows dynamic states for a chaotic system with 4 state variables depending on the sign of the Lyapunov exponent [26]. In this table $L_i, i = 1,2,3,4$ represents the ith Lyapunov exponent of the system.

Table 1. dynamic states of a chaotic system with 4 state variables depending on Lyapunov exponents

Dynamic behavior type	L_1	L_2	L_3	L_4
Equilibrium point	–	–	–	–
Limit cycle	0	–	–	–
Semi-periodic	0	0	–	–
Chaotic behavior	+	0	–	–
High-order chaos	+	+	0	–

Lyapunov exponent of system (1) is specified in (7). As can be seen, the system has two positive Lyapunov exponents, a zero exponent and a negative Lyapunov exponent. According to Table (1), this case presents high-order chaos in system.

$$\begin{cases} L_1 = 2.2316 \\ L_2 = 0.59014 \\ L_3 = 0 \\ L_4 = -14.4994 \end{cases} \tag{7}$$

3. Creating different dynamic behaviors for high-order chaotic system by changing parameter a.

Changing one of system parameters and keeping the rest constant, the system would show different dynamic behaviors. For example, if parameter a is changed in the range, the high-order system (1) shows the behavior as in Table 2.

Table 2. Different dynamic behaviors of the system in terms of changes of parameter ***a***

Range of parameter *a*	Type of system dynamic behavior
$0 \le a \le 0.3$	Low-order chaos
$0.4 \le a \le 0.9$	Limit cycle (periodic)
$1 \le a \le 1.4$	semi-periodic
$1.5 \le a \le 3.5$	Limit cycle (periodic)
$3.7 \le a \le 3.9$	semi-periodic
$4.3 \le a \le 15$	High-order chaos

4. High-order chaos control for chaotic systems

Consider equations of a high-order nonlinear continuous chaotic system as follows:

$$\begin{cases} \dot{X} = f(X,t) \\ X(0) = X_0 \in \Re^n \end{cases} \tag{8}$$

Where X is considered as $X = (x, y, z, w)$ vector. LQR controller is designed in such a way that the controlled system $\dot{X} = f(X,t) + u$ converges to the unstable equilibrium point or its own periodic paths. If $u = 0$ then controlled system becomes the primary chaotic system. The proposed controller is added to the second equation of the chaotic system. System under study is a chaotic system with the following differential equation:

$$\begin{cases} \dot{x} = 7.7(y-x) - yz^2 \\ \dot{y} = 8x + 4xz^2 + 8w + u \\ \dot{z} = -4z + y^2 + xw \\ \dot{w} = -2y \end{cases} \tag{9}$$

LQR controller is a linear controller that is applied to a linear system. For this reason, in order to utilize this controller, the system should be linearized around the desired operating point. The desired operating point is the origin and the system is linearized around this point as follows:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial w} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial w} \end{bmatrix} = \begin{bmatrix} -7.7 & 7.7 & 0 & 0 \\ 8 & 0 & 0 & 8 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \ , \ B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \\ \frac{\partial f_4}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Next, LQR controller is designed for linearized system and is applied on nonlinear system. This controller is designed to optimize the following criteria.

$$J = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)dt$$

The controller parameters are chosen as follows:

$$Q = 100 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ , \ R = 10 \ , \ N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Simulation results are shown in Figure 8. As shown in this figure, the controlled system is converged toward the origin. To compare the proposed controller with other controllers, one can refer to Figure 9. This figure demonstrates the state variables of the system (9) considering that the origin of the system is stabilized using linear state feedback controller. It can be said that the proposed controller has improved a bit in terms of time to reach the final value, also the range in which state variables change becomes smaller.

CONCLUSION

In this paper, a high-order chaotic system with an equilibrium point was proposed and by investigating some criteria such as energy dissipation and Lyapunov exponents, the existence of chaos was proven analytically. On the other hand, by investigating response time of the system and also system exponents, the existence of high-order chaos in system was also proven through simulations. Then, by changing one of system parameters and keeping the rest of the parameters constant, a different dynamic behavior of a chaotic system was observed.

Finally, an LQR controller was designed for the linearized dynamic system and by applying it to the initial nonlinear system, optimal performance of the proposed controller was shown in the stabilization of nonlinear system around its equilibrium point. Also the proposed controller compared to the linear state feedback controller is better in terms of the time to reach the final value and the range of state variables.

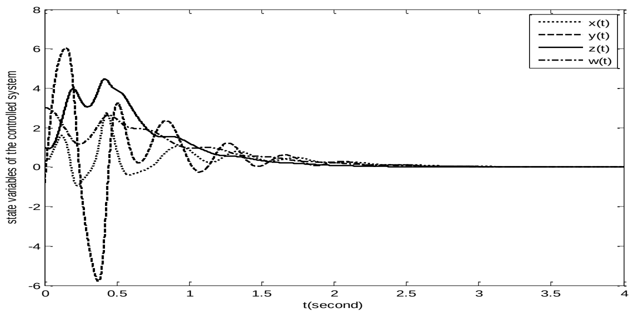

Figure 8. Response time of 4 state variables of the system controlled by LQR controller

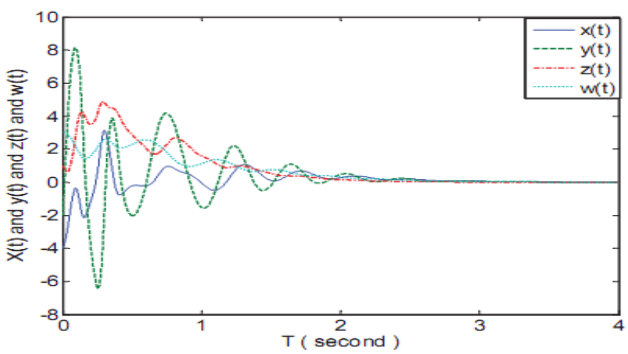

Figure 9. Response time of 4 state variables of system controlled by linear state feedback controller with K = -30

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