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JWEET

Correction to a Rational Approximation of a Fractional-Order Laplacian Operator

Kenneth V. Cartwright*, Jamaal Mesidor and Krystan Russell

College of The Bahamas, P.O. Box N4912, Nassau, Bahamas

*Corresponding's author's Email: kvcartwright@yahoo.com

***Abstract** – Rational approximations of a fractional-order Laplacian Operator are needed in the design of fractional-order differentiators, integrators and filters. Unfortunately, a third-order approximation of such is given incorrectly in at least two publications. We point out and correct this error. We also provide magnitude and phase plots which demonstrate that our correction is indeed valid.*

***Keywords**: Fractional Differentiator, Fractional-Order Operator, Analog Fractance Approximation, Fractance Device.*

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INTRODUCTION

In order to design analogue fractional differentiators, integrators and filters with conventional capacitors, resistors and inductors, the Laplacian Operator s^{α} must be approximated by rational functions of s, where $s = j\omega,\ j = \sqrt{-1}$ and ω (rad/s) is the angular frequency of the applied sinusoid [1, 2]. Table 2 of Krishna [3] study provides such approximations for various degrees of accuracy for $0 \le \alpha \le 1$. Unfortunately there is an error in this table that has also been repeated in Table 2 of Tanwar and S. Kumar [4] work. In this letter, we wish to point out and correct this error.

Error in Publications

Table 1 of [4] and Table 1 of [5] state that a third-order approximation to $\sqrt{s}$ is given by

$$\sqrt{s} \approx \frac{7s^3 + 35s^2 + 21s + 1}{s^3 + 21s^2 + 35s + 7}. \qquad (1)$$

On the other hand, Table 2 of [3] and Table 2 of [4] give

$$s^{\alpha} \approx \frac{P_0 s^3 + P_1 s^2 + P_2 s + P_3}{Q_0 s^3 + Q_1 s^2 + Q_2 s + Q_3}, \qquad (2)$$

where

$$\begin{aligned} P_0 &= Q_3 = \alpha^3 + 6\alpha^2 + 11\alpha + 6 \\ P_1 &= Q_2 = -3\alpha^3 - 6\alpha^2 + 27\alpha + 54 \\ P_2 &= Q_1 = 3\alpha^3 - 6\alpha^2 + 27\alpha + 54 \\ P_3 &= Q_0 = -\alpha^3 + 6\alpha^2 - 11\alpha + 6. \end{aligned}$$

Substituting $\alpha = 1/2$ into (2) should produce (1). Also, substituting $\alpha = 1$ into (2) should produce s on the right-hand-side of (2). Unfortunately, neither of these happens because there is an error in P_2 and Q_1 which should be corrected to be

$$P_2 = Q_1 = 3\alpha^3 - 6\alpha^2 - 27\alpha + 54. \qquad (3)$$

Using (3) in (2), both aforementioned substitutions produce the desired answers.

To further demonstrate that the suggested correction is valid, we use (3) to plot magnitude (in $\mathrm{dB} = 20\log_{10}\left|s^{\alpha}\right|$) in Fig. 1(a) and phase of (2) in Fig. 1(b), for $\alpha = 0.1$.

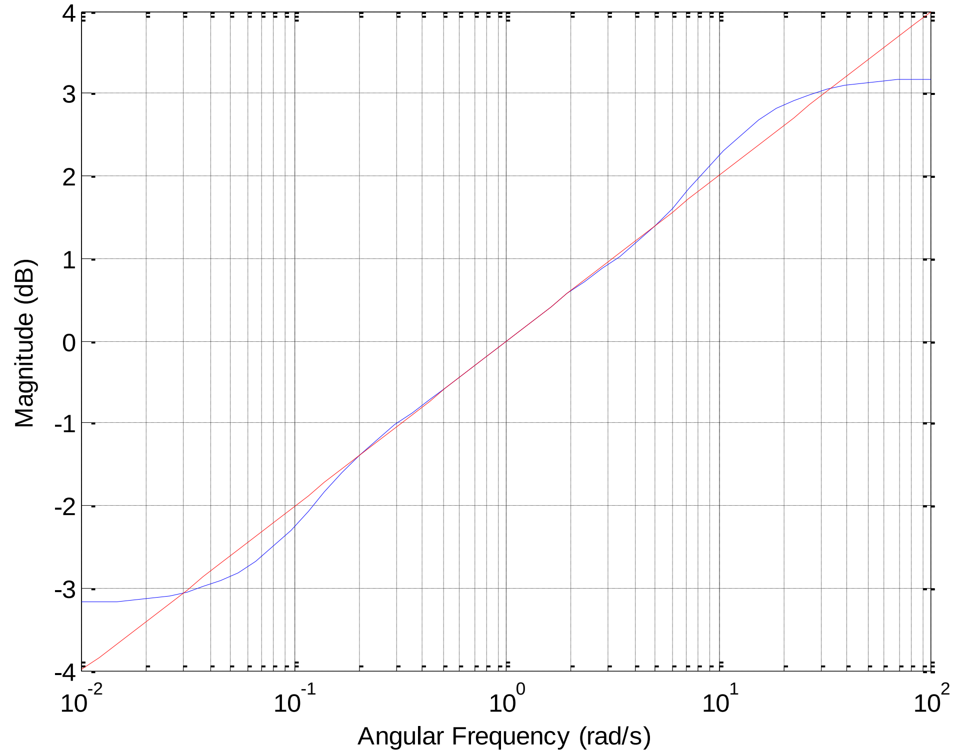

Figure 1(a). Plot of magnitude of (2) using (3) for $a = 0.1$.

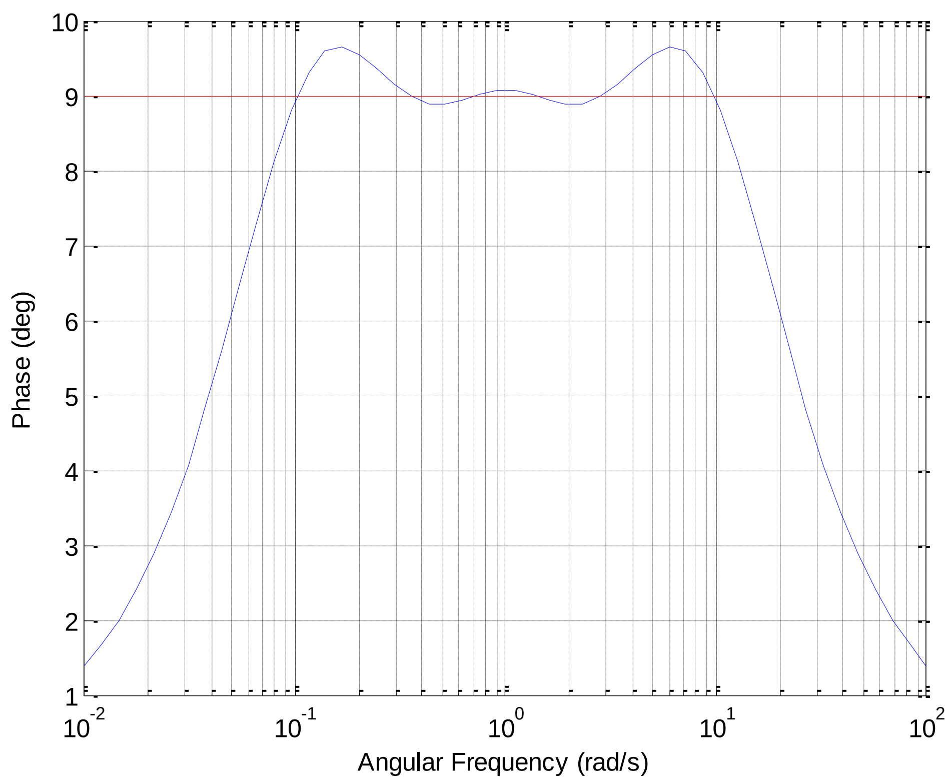

Figure 1(b). Plot of phase of (2) using (3) for $\alpha = 0.1$.

Note that the straight lines in Fig. 1(a) and Fig. 1(b) are the theoretical values; i.e., the slope of the straight-line for the magnitude is $\alpha 20\text{dB/decade}$ and the phase angle is constant at $\alpha 90^o$. As can be seen from Fig. 1(a) and Fig. 1(b), the approximations of (2) are quite good if (3) is used. However, if these plots are redone with the original expression for $P_2 = Q_2$, it will be seen that the approximations are quite poor.

CONCLUSION

We have pointed out and corrected an error in Table 2 of Krishna [3] study, which was also repeated in Table 2 of [4]. This correction should be of benefit to designers of analogue fractional-order differentiators, integrators and filters.

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